Philosophy 211

Take Home 2 – Due Thursday, December 14th

P1. $\exists x((Sx \,\&\, \forall y(Ty \rightarrow {\sim}Pxy)) \,\&\, \forall z((Sz \,\&\, x \neq z) \rightarrow \exists u(Tu \,\&\, Pzu)))$
P2. $\forall x(Tx \rightarrow \exists y(Sy \,\&\, Pyx))$
P3. $\forall z(Sz \rightarrow \exists y(Ty \,\&\, {\sim}Pzy))$
P4. $\forall x(Tx \rightarrow {\sim}\exists y\exists z(((Sy \,\&\, Sz) \,\&\, (Pyx \,\&\, Pzx)) \,\&\, y \neq z))$
P5. $\forall x\forall y\forall z(((Tx \,\&\, Ty) \,\&\, (x \neq y \,\&\, Sz)) \rightarrow ({\sim}Pzx \vee {\sim}Pzy))$
P6. $\forall x((Sx \,\&\, \exists y(Ty \,\&\, Pxy)) \rightarrow \forall z\forall u(((Tz \,\&\, Tu) \,\&\, u \neq z) \rightarrow (Pxz \vee Pxu))$

I. Which of P1-P6 are true in the following Diagrams? (18)

Diagram 1

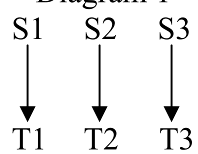


Diagram 2

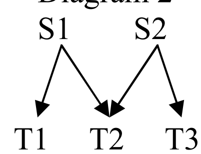
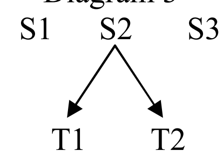


Diagram 3



II. Show that {P1-P6} is consistent by providing a model or constructing a diagram that makes all six sentences true. (8)

III. Prove the following claims from only P1-P6: (24)

(6) A. $\exists x\exists y(Tx \,\&\, Ty \,\&\, x \neq y)$

(8) B. $\exists x\exists y\exists z(Sx \,\&\, Sy \,\&\, Sz \,\&\, x \neq y \,\&\, x \neq z \,\&\, y \neq z)$

(10) C. $\forall x\forall y\forall z((Tx \,\&\, Ty \,\&\, Tz) \rightarrow (x=y \vee x=z \vee y=z))$

BONUS QUESTION (10) D. $\forall x\forall y\forall z\forall w((Sx \,\&\, Sy \,\&\, Sz \,\&\, Sw) \rightarrow (x=y \vee x=z \vee x=w \vee y=z \vee y=w \vee z=w))$

You could prove each claim separately, but many of your lines would be repeated and you would have to do multiple existential eliminations multiple times. Since this is true:

A&C is equivalent to: $\exists x\exists y(Tx \,\&\, Ty \,\&\, x \neq y \,\&\, \forall z(Tz \rightarrow (z=x \vee z=y)))$
B&D is equivalent to: $\exists x\exists y\exists z(Sx \,\&\, Sy \,\&\, Sz \,\&\, x \neq y \,\&\, x \neq z \,\&\, y \neq z \,\&\, \forall w(Sw \rightarrow (w=x \vee w=y \vee w=z)$

You can prove A, B, C, and D by proving these last two claims instead.